

Thermodynamic Study for Conformal Phase in Large N_f Gauge Theory

Kohtaroh Miura*

INFN Laboratori Nazionali di Frascati, I-00044, Frascati (RM), Italy

E-mail: Kohtaroh.Miura@lnf.infn.it

Maria Paola Lombardo

INFN Laboratori Nazionali di Frascati, I-00044, Frascati (RM), Italy

Humboldt-Universität zu Berlin, Institut für Physik, D-12489 Berlin, Germany

E-mail: Mariapaola.Lombardo@lnf.infn.it

Elisabetta Pallante

Centre for Theoretical Physics, University of Groningen, 9747 AG, Netherlands

E-mail: e.pallante@rug.nl

We investigate the chiral phase transition at finite temperature (T) in colour $SU(N_c = 3)$ Quantum Chromodynamics (QCD) with six species of fermions ($N_f = 6$) in the fundamental representation [1]. The simulations have been performed by using lattice QCD with improved staggered fermions. The critical couplings β_L^c for the chiral phase transition are observed for several temporal extensions N_t , and the two-loop asymptotic scaling of the dimensionless ratio T_c/Λ_L (Λ_L = Lattice Lambda-parameter) is found to be achieved for $N_t \geq 6$. Further, we collect β_L^c at $N_f = 0$ (quenched), and $N_f = 4$ at a fixed $N_t = 6$ as well as $N_f = 8$ at $N_t = 6, 12$, the latter relying on our earlier study. The results are consistent with enhanced fermionic screening at larger N_f . The ratio T_c/Λ_L depends very mildly on N_f in the $N_f = 0 - 4$ region, begins increasing at $N_f = 6$, and significantly grows up at $N_f = 8$, as N_f reaches to the edge of the conformal window. We discuss the interrelation of the results with preconformal dynamics in the light of a functional renormalization group analysis.

The XXIX International Symposium on Lattice Field Theory - Lattice 2011

July 10-16, 2011

Squaw Valley, Lake Tahoe, California

*Speaker.

1. Introduction

Emergence of a conformal symmetry and a preconformal (walking) behavior in strongly flavored non-Abelian gauge theories has received much attention. Walking dynamics near the infrared fixed point has been advocated as a basis for strongly interacting mechanisms of electroweak symmetry breaking. Lattice Monte-Carlo simulations are expected to provide a solid theoretical base to understand the (pre-)conformal nature in the gauge theory.

A second zero of the two-loop beta-function of massless QCD with N_f flavours implies, at least perturbatively, the appearance of an infrared fixed point (IRFP) at $N_f \gtrsim 8.05$ [2] with the restoration of conformal symmetry before the loss of asymptotic freedom (LAF) at $N_f^{\text{LAF}} = 16.5$. Conformality should emerge when the renormalized coupling at the would be IRFP is not strong enough to break chiral symmetry. This condition provides the lower bound N_f^c of a so called conformal window in the flavor space, and we find elaborated analytic predictions [3, 4]: for instance, the functional renormalization group method [5] suggests $N_f^c \sim 12$. Before the emergence of conformal symmetry, a qualitative change of dynamics is claimed at $N_f = 6$ based on instanton study [6].

Recent lattice studies[7] focused on the computation of the edge of the conformal window N_f^c and the analysis of the conformal window itself, either with fundamental fermions [8 – 15]. or other representations [16]. Among the many interesting results with fundamental fermions, we single out the observation that QCD with three colours and eight flavours is still in the hadronic phase [9, 10], while $N_f = 12$ seems to be close to N_f^c , with some groups favouring conformality [8, 9, 11, 12], and others chiral symmetry breaking [14]. The onset of new strong dynamics at $N_f = 6$ has been implied via an enhancement of the ratio of chiral condensate to cubed pseudoscalar decay constant [17].

Using the thermal transition as a tool for investigating preconformal dynamics has been largely inspired by a renormalization group analysis [5]. The critical temperature for the chiral phase transition has been obtained as a function of N_f . Then the onset of the conformal window has been estimated by locating the vanishing critical temperature. The phase transition line is almost linear with N_f for small N_f , and clearly elucidates the universal critical behaviour at zero and non-zero temperature in the vicinity of N_f^c . Thus, it would be a promising direction to extend the knowledge of finite T lattice QCD to the larger N_f region, by using the FRG results as analytic guidance.

In this proceedings, we investigate the thermal chiral phase transition for $N_f = 6$ colour $SU(N_c = 3)$ QCD by using lattice QCD Monte Carlo simulations with improved staggered fermions based on our recent study [1]. $N_f = 6$ is expected to be in the important regime as suggested by the results in Refs. [6, 8]. We also compute the critical couplings for $N_f = 0$ (quenched) and $N_f = 4$ at $N_t = 6$, and use the results from Ref. [10] for $N_f = 8$. Then we investigate N_f dependences of the chiral phase transition.

2. Simulation setups

Simulations have been performed in the same as in the study used for $N_f = 8$ in Ref. [10]: We have utilized the publicly available MILC code [18] with the use of an improved version of the staggered action, the Asqtad action, with a one-loop Symanzik [19, 20] and tadpole [22] improved

gauge action. The tadpole factor u_0 is determined by performing zero temperature simulations on the 12^4 lattice, and used as an input for finite temperature simulations.

To generate configurations with mass degenerate dynamical flavours, we have used the rational hybrid Monte Carlo algorithm (RHMC) [21]. Simulations for $N_f = 6$ have been performed by using two pseudo-fermions, and subsets of trajectories for the chiral condensates and Polyakov loop have been compared with those obtained by using three pseudo-fermions with the same Monte Carlo time step $d\tau$ and total time length τ of a single trajectory. We have observed very good agreement between the two cases for both evolution and thermalization. We have monitored the Metropolis acceptance and reject ratio, and adjusted $\tau = 0.2 - 0.24$ and $d\tau = 0.008 - 0.016$ to realize the best performance.

Measured observables are the expectation values of the chiral condensate and Polyakov loop,

$$a^3 \langle \bar{\psi} \psi \rangle = \frac{N_f}{4N_s^3 N_t} \left\langle \text{Tr}[\mathbf{M}^{-1}] \right\rangle, \quad L = \frac{1}{N_c N_s^3} \sum_{\mathbf{x}} \text{Re} \left\langle \text{tr}_c \prod_{t=1}^{N_t} U_{4,t\mathbf{x}} \right\rangle, \quad (2.1)$$

where N_s (N_t) represents the number of lattice sites in the spatial (temporal) direction, $U_{4,t\mathbf{x}}$ is the temporal link variable, and tr_c denotes the trace in colour space. The output of this measurement is the critical coupling β_L^c for the chiral phase transition.

3. Results

All results have been obtained for a fermion bare lattice mass $am = 0.02$. In the left panel of Figs. 1, the expectation values of the chiral condensate $a^3 \langle \bar{\psi} \psi \rangle$ are displayed as a function of β_L for several N_t . It is found that different N_t give a different behaviour of $a^3 \langle \bar{\psi} \psi \rangle$. The asymptotic scaling analysis below will confirm that it corresponds to a thermal chiral phase transition (or crossover) in the continuum limit.

All values of the critical lattice coupling β_L^c are summarized in Table 1. For larger N_t , the signal for the chiral phase transition becomes less clear, hence we investigate the histogram of the chiral condensate: The histogram for $N_t = 8$ exhibits the double-peak structure at $\beta_L = 5.2$, *i.e.*, the competition between chirally symmetric and broken vacua. The critical coupling can be estimated as $\beta_L^c = 5.225(25)$ for $N_t = 8$. For $N_t = 12$, we also observe the double-peak structure in the histogram of the chiral condensate around $\beta_L = 5.45$.

These results can be analyzed and interpreted in terms of the two-loop asymptotic scaling. Let us consider the two-loop lattice beta function,

$$\beta(g) = -(b_0 g^3 + b_1 g^5), \quad (3.1)$$

$$(b_0, b_1) = ((11 - 2N_f/3)/(4\pi)^2, (102 - 38N_f/3)/(4\pi)^4), \quad (3.2)$$

for fundamental fermions in colour SU(3). From Eq. (3.1), we obtain the well known two-loop asymptotic scaling,

$$\Lambda_L a(\beta_L) = (2N_c b_0 / \beta_L)^{-b_1/(2b_0^2)} \exp[-\beta_L / (4N_c b_0)]. \quad (3.3)$$

Here, Λ_L is the so-called lattice Lambda-parameter, and $\beta_L = 2N_c/g^2$, with $g = \sqrt{2N_c/10} \cdot g_L$. This definition effectively takes account of the improvement of the staggered lattice action when comparing to the asymptotic scaling law, see Ref. [10]. We insert Λ_L to the definition of temperature

$$T \equiv [a(\beta_L)N_t]^{-1},$$

$$N_t^{-1} = (T_c/\Lambda_L) \times (\Lambda_L a(\beta_L^c)) , \quad (3.4)$$

and extract the physical quantity T_c/Λ_L by substituting the simulation outputs β_L^c for Eq. (3.4). This ratio must be unique as long as the asymptotic scaling Eq. (3.3) is verified for a given β_L^c .

In the right panel of Fig. 1, the slope of the line connecting the origin and the data points corresponds to T_c/Λ_L . The $N_t = 6, 8$, and 12 points have a common slope to a very good approximation, while the $N_t = 4$ result falls on a smaller slope. The latter is interpreted as a scaling violation effect due to the use of a too small N_t . The existence of a common T_c/Λ_L for $N_t \geq 6$ indicates that the data are consistent with the two-loop asymptotic scaling Eq. (3.3), confirms the thermal nature of the transition and that $N_f = 6$ is outside the conformal window, as expected from a previous $N_f = 8$ study [10]. A linear fit provides $T_c/\Lambda_L = 1.02(12) \times 10^3$, which can be interpreted as the value in the continuum limit for $N_f = 6$ QCD.

In order to have a more complete overview, we have performed simulations for the theory with $N_f = 0$ (quenched) and $N_f = 4$, only at $N_t = 6$. These theories are of course very well investigated, however we have not found in the literature results for the same action as ours. We note that in a previous lattice study with improved staggered fermions [23], asymptotic scaling was observed for $N_t \geq 6$ for $0 \leq N_f \leq 4$. Table 1 shows a summary of our results for the critical coupling β_L^c of the chiral phase transition at finite temperature for $N_f = 0, 4, 6$, and 8 - the latter from Ref. [10].

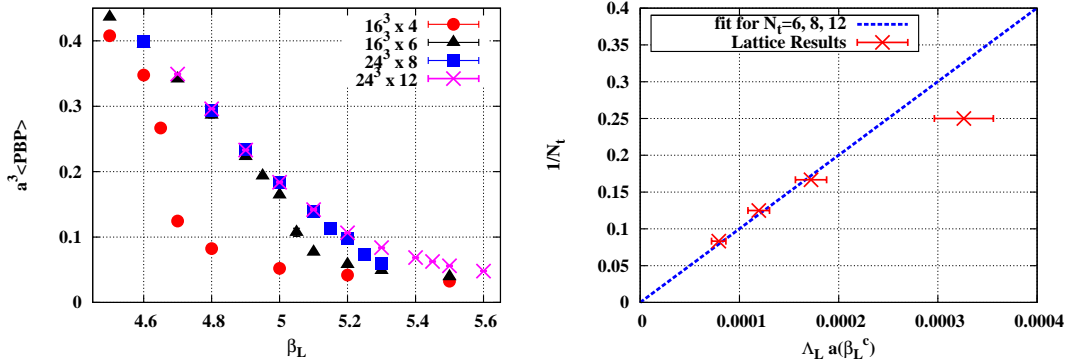


Figure 1: Left: The chiral condensate $a^3\langle\bar{\psi}\psi\rangle$ for $N_f = 6$ and $am = 0.02$ in lattice units, as a function of β_L , for $N_t = 4, 6, 8$, and 12. Error-bars are smaller than symbols. Right: The thermal scaling behaviour of the critical lattice coupling β_L^c . Data points for $\Lambda_L a(\beta_L^c)$ at a given $1/N_t$ are obtained by using β_L^c from Table 1 as input for extracting $\Lambda_L a(\beta_L^c)$ in the two-loop expression Eq. (3.3). The dashed line is a linear fit with zero intercept to the data with $N_t > 4$.

In the left panel of Fig. 2, we display the critical values of the lattice coupling $g_c = \sqrt{2N_c/\beta_L^c}$ from Table 1 in the Miransky-Yamawaki phase diagram. Consider the $N_t = 6$ results: it is expected that an increasing number of flavours favors chiral symmetry restoration. Indeed, we find that, on a fixed lattice, the critical coupling increases with N_f in agreement with early studies and naive reasoning. The precise dependence of the critical coupling on N_f at fixed N_t is not known. It is, however, amusing to note that the results seem to be smoothly connected by an almost straight line: the brown line in the plot is a linear fit to the data. Comparing the trend for $N_f = 6$ to the one for

$N_f = 8$ for varying N_t , one can infer a decreasing in magnitude (and small) step scaling function, hence a walking behaviour. Further study is needed at larger N_f , and by using the same action used for $N_f = 0 - 8$, to confirm or disprove it.

Table 1: Summary of the critical lattice couplings β_L^c for the theories with $N_f = 0, 4, 6, 8$, $am = 0.02$ and varying $N_t = 4, 6, 8, 12$. All results are obtained using the same lattice action.

$N_f \backslash N_t$	4	6	8	12
0	-	7.88 ± 0.05	-	-
4	-	5.89 ± 0.03	-	-
6	4.675 ± 0.025	5.025 ± 0.025	5.225 ± 0.025	5.45 ± 0.05
8	-	4.1125 ± 0.0125	-	4.34 ± 0.04

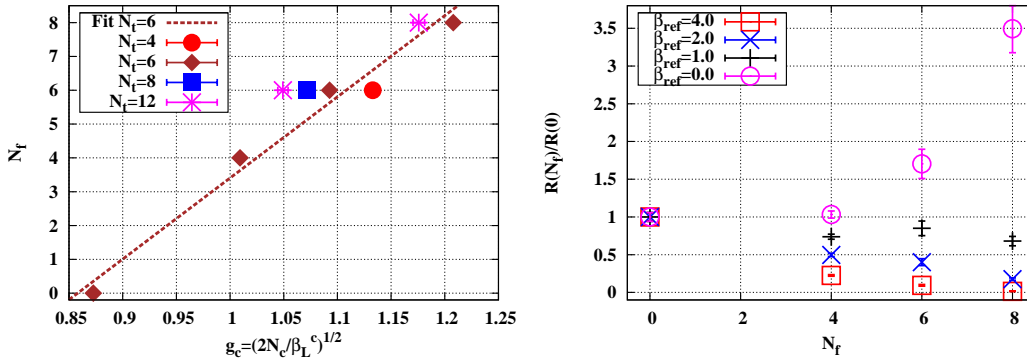


Figure 2: Left: Critical values of the lattice coupling $g_c = \sqrt{2N_c/\beta_L^c}$ for theories with $N_f = 0, 4, 6, 8$ and for several values of N_t in the Miransky-Yamawaki phase diagram. The dashed (brown) line is a linear fit to the $N_t = 6$ results. Right: The N_f dependence of $R(N_f)/R(0)$ for several finite fixed β_L^{ref} . Here, $R(N_f) \equiv (T_c/\Lambda_{\text{ref}})(N_f)$.

Next, we study the N_f dependence of the ratio T_c/Λ_L and related quantities. In addition to the scale Λ_L , we introduce more UV reference energy scale Λ_{ref} , which is associated with a reference coupling β_L^{ref} . Then Eq. (3.3) is generalized as

$$\Lambda_{\text{ref}}(\beta_L^{\text{ref}}) a(\beta_L) = \left(\frac{b_1}{b_0^2} \frac{\beta_L + 2N_c b_1/b_0}{\beta_L^{\text{ref}} + 2N_c b_1/b_0} \right)^{b_1/(2b_0^2)} \exp \left[-\frac{\beta_L - \beta_L^{\text{ref}}}{4N_c b_0} \right]. \quad (3.5)$$

At leading order of perturbation theory $b_1 \rightarrow 0$, we find $\Lambda_{\text{ref}}/\Lambda_L = \exp[\beta_L^{\text{ref}}/(4N_c b_0)]$. This equation would be analogous of the ratio $\Lambda_L/\Lambda_{\text{MS}}$ derived in [24] for Wilson fermions up to a further linear dependence on N_f in the numerator of the exponent. In a nutshell, the difference originates from the fact that we are fixing a bare reference coupling β_L^{ref} , which will be specified later. Notice that by construction Λ_{ref} reproduces the lattice Lambda-parameter Λ_L in the limit $\Lambda_{\text{ref}}(\beta_L^{\text{ref}} \rightarrow 0) = \Lambda_L(1 + \mathcal{O}(1/\beta_L^c))$.

Let us consider first $R(N_f)|_{\beta_L^{\text{ref}}=0.0} = T_c/\Lambda_L$. The values of T_c/Λ_L are found to be 600 ± 34 , 620 ± 28 , 1023 ± 117 , and 2098 ± 191 for $N_f = 0, 4, 6$, and 8 , respectively, and represented as circles in the right panel of Fig. 2 (the vertical axis is normalized by $R(0) = (T_c/\Lambda_L)(N_f = 0)$ for each β_L^{ref}). The ratio does not show a significant N_f dependence in the region $0 \leq N_f \leq 4$, it starts increasing at $N_f = 6$, and undergoes a rapid rise around $N_f = 8$. The nearly constant nature of T_c/Λ_L in the region $N_f \leq 4$ indicates that the role of such energy scale is not significantly changed by the variation of N_f (see [25] for a detailed discussion of this point.) In turn, the increase of T_c/Λ_L in the region $N_f \geq 6$ might well imply that the chiral dynamics becomes different from the one for $N_f \leq 4$. Indeed, a recent lattice study [17] indicates that $N_f = 6$ is close to the threshold for preconformal dynamics.

We now consider T_c/Λ_{ref} with finite β_L^{ref} . The N_f dependence of the ratio $R(N_f) \equiv (T_c/\Lambda_{\text{ref}})(N_f)$ is shown for several β_L^{ref} in the right panel of Fig. 2, (with normalization by $R(0) = (T_c/\Lambda_{\text{ref}})(N_f = 0)$ for each β_L^{ref}). T_c/Λ_{ref} is now a decreasing function of N_f for a larger β_L^{ref} . The Λ_{ref} associated with a $\beta_L^{\text{ref}} \gg \beta_* = 2N_c/g_{\text{IRFP}}^2$ would be less sensitive to the IR or chiral dynamics. Assuming $N_f^c \simeq 12$, the two-loop beta-function leads to $\beta_* = -2N_c b_1/b_0 \simeq 0.63$. The decreasing nature of $(T_c/\Lambda_{\text{ref}})(N_f)$ is found to start around $\beta_L^{\text{ref}} = 1.0 \gtrsim \beta_*$. Thus, the use of a UV reference scale leads to the decreasing $(T_c/\Lambda_{\text{ref}})(N_f)$. This trend is consistent with the FRG study [5], where the decreasing $T_c(N_f)$ has been obtained by using the τ -lepton mass m_τ as a common UV reference scale with a common coupling $\alpha_s(m_\tau)$. We note that we have constrained our analyses $\beta_L^{\text{ref}} < \beta_{\text{UV}} = \beta_L^c(N_f) \leq 4.1125 \pm 0.0125$.

With the use of a UV reference scale, we should observe the predicted critical behavior [5],

$$T_c(N_f) = K|N_f - N_f^c|^{-1/\theta}. \quad (3.6)$$

By choosing the critical exponent θ in the range predicted by FRG: $1.1 < 1/|\theta| < 2.5$, our data are consistent with the values $N_f^c = 9(1)$ for $\beta_L^{\text{ref}} = 4.0$ and $N_f^c = 11(2)$ for $\beta_L^{\text{ref}} = 2$. We plan to extend and refine this analysis in the future, and here we only notice a reasonable qualitative behaviour.

4. Summary

We have studied the chiral phase transition at finite T for colour $SU(3)$ QCD with $N_f = 6$ by using lattice QCD Monte-Carlo simulations with improved staggered fermions [1]. We have determined the critical lattice coupling β_L^c for several lattice temporal extensions N_t , and extracted the dimensionless ratio T_c/Λ_L (Λ_L = Lattice Lambda-parameter) by using two-loop asymptotic scaling. The analogous result for $N_f = 8$ has been extracted from Ref. [10]. T_c/Λ_L for $N_f = 0$ and $N_f = 4$ has been measured at fixed $N_t = 6$, barring asymptotic scaling violations. Then we have discussed the N_f dependence of the ratios T_c/Λ_L and T_c/Λ_{ref} , where Λ_{ref} is a UV reference energy scale, related to Λ_L via $\Lambda_{\text{ref}}/\Lambda_L \simeq \exp[\beta_L^{\text{ref}}/(4N_c b_0)]$. We have observed that T_c/Λ_L shows an increase in the region $N_f = 6 - 8$, while it is approximately constant in the region $N_f \leq 4$. We have discussed this qualitative change for $N_f \geq 6$ and a possible relation with a preconformal phase. The ratio T_c/Λ_{ref} is a decreasing function of N_f . This behaviour is consistent with the result obtained in the functional renormalization group analysis [5]. Next steps of the current project involve a scale setting at zero temperature by measuring a common UV observable.

Acknowledgements

We thank Holger Gies, Jens Braun, Michael Müller-Preussker, Marc Wagner, Biagio Lucini, Volodya Miransky, Albert Deuzeman, and Tiago Nunes da Silva for fruitful discussions. This work was in part based on the MILC Collaboration's public lattice gauge theory code [18]. The numerical calculations were carried out on the IBM-SP6 at CINECA, Italian-Grid-Infrastructures in Italy, and the Hitachi SR-16000 at YITP, Kyoto University in Japan.

References

- [1] K. Miura, M. P. Lombardo and E. Pallante, arXiv:1110.3152 [hep-lat].
- [2] W. E. Caswell, Phys. Rev. Lett. **33** (1974) 244; T. Banks and A. Zaks, Nucl. Phys. B **196** (1982) 189.
- [3] V. A. Miransky and K. Yamawaki, Phys. Rev. D **55** (1997) 5051 [Erratum-ibid. D **56** (1997) 3768].
- [4] See Ref. [7], where the analytic results are summarized with references.
- [5] J. Braun, C. S. Fisher, H. Gies, Phys. Rev. **D84** (2011) 034045; J. Braun and H. Gies, JHEP **1005** (2010) 060; **0606** (2006) 024.
- [6] M. Velkovsky and E. V. Shuryak, Phys. Lett. B **437** (1998) 398.
- [7] For recent reviews, see L. Del Debbio, PoS **LATTICE2010** (2010) 004; E. Pallante, PoS **LATTICE2009** (2009) 015.
- [8] T. Appelquist, G. T. Fleming, M. F. Lin, E. T. Neil, D. A. Schaich, Phys. Rev. **D84**, 054501 (2011).
- [9] T. Appelquist, G. T. Fleming and E. T. Neil, Phys. Rev. D **79** (2009) 076010; Phys. Rev. Lett. **100** (2008) 171607 [Erratum-ibid. **102** (2009) 149902].
- [10] A. Deuzeman, M. P. Lombardo and E. Pallante, Phys. Lett. B **670** (2008) 41.
- [11] A. Deuzeman, M. P. Lombardo and E. Pallante, Phys. Rev. D **82** (2010) 074503.
- [12] A. Hasenfratz, Phys. Rev. D **82** (2010) 014506.
- [13] A. Hasenfratz, Phys. Rev. D **80** (2009) 034505.
- [14] Z. Fodor, K. Holland, J. Kuti, D. Negradi and C. Schroeder, Phys. Lett. **B703** (2011) 348-358.
- [15] Z. Fodor, K. Holland, J. Kuti, D. Negradi and C. Schroeder, Phys. Lett. B **681** (2009) 353.
- [16] See reviews in Ref. [7], and references are therein.
- [17] T. Appelquist *et al.*, Phys. Rev. Lett. **104** (2010) 071601.
- [18] MILC Collaboration, <http://www.physics.indiana.edu/~sg/milc.html>
- [19] C. Bernard *et al.*, Phys. Rev. D **75** (2007) 094505.
- [20] M. Luscher and P. Weisz, Phys. Lett. B **158** (1985) 250; Commun. Math. Phys. **97** (1985) 59 [Erratum-ibid. **98** (1985) 433].
- [21] M. A. Clark, PoS **LAT2006** (2006) 004.
- [22] G. P. Lepage, P. B. Mackenzie, Phys. Rev. D **48** (1993) 2250.
- [23] S. Gupta, Phys. Rev. D **64** (2001) 034507.
- [24] H. Kawai, R. Nakayama and K. Seo, Nucl. Phys. B **189** (1981) 40.
- [25] J. Braun, Phys. Rev. **D81** (2010) 016008.